Analysis and design of a global discrete and non-linear PD controller for robot manipulator

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Resumen

En este artículo se analiza y diseña un controlador PD discreto y no lineal con compensación de la dinámica del robot manipulador en la trayectoria deseada para minimizar los efectos producidos por las perturbaciones acopladas; el objetivo de control se alcanza calculando la cota mínima que garantiza condiciones suficientes para la existencia y unicidad del punto de equilibrio y la cota mínima que garantiza condiciones suficientes para lograr la estabilidad asintótica en forma global del sistema en lazo cerrado, a través de la utilización del método directo de Lyapunov, derivado de este conjunto de desigualdades se procede al diseño del controlador PD en el dominio del tiempo continuo, y posteriormente este controlador se transforma al dominio del tiempo discreto por el método numérico de Euler. El rendimiento y el resultado se demuestran mediante un caso práctico con el robot manipulador Niryio One.

Palabras clave—Estabilidad de Lyapunov, Control discreto no lineal, Robot manipulador.

Abstract

In this paper, we analyze and design a discrete and non-linear PD controller with compensation of the dynamics of the robot manipulator in the desired trajectory to minimize the effects produced by matched disturbances; the objective is reached by computing the minimum bound that guarantees sufficient conditions to existence and uniqueness of the equilibrium point and the minimum bound that guarantee sufficient conditions to achieve global asymptotic stability of the closed-loop system, through direct method of Lyapunov, derived from these inequalities we proceed to the design of the PD controller in the continuo-time domain, this controller is transformed to the discrete-time domain by the Euler numerical method. The performance and result are demonstrated by practice-case with the manipulator robot Niryio One.

Keywords— *Lyapunov Stability, Nonlinear Discrete PD Control, Robot Manipulator*

1. INTRODUCTION

A robot manipulator is a non-linear system and has complicated behavior that includes interactions between multiple joints, non-linear effects such as centrifugal and Coriolis forces, variable inertia depending on arm configuration, and as higher performance is pursued in terms of speed and precision, these dynamics become more important [1], other factors that affect the performance of manipulator robots are the effects inherent to mechanical systems such as the effect of elasticity in the mechanical coupling between the actuator and the joint, friction, angular play between gears, dead zone caused by use of gears, cyclo reductors, harmonic reductors, toothed bands, chains, cables, endless screws, in addition to exogenous disturbances such as torques or forces due to possible physical contact with the robot body or interaction with external objects and finally the uncertainty in the model.

There are excellent research works where problems related to the performance of manipulator robots are addressed, for example in [2], [3], [4], and [5] the designs of control systems that ensure global asymptotic stability for disturbance-free manipulator robots. Regarding robust controllers applied to robots that operate under uncertain conditions and report asymptotic stability conditions globally and locally, it is reported by [6], [7], [8], and [9]. In recent years, numerous works have been reported where they successfully address the aforementioned problems that affect manipulator robots, each of these methodologies attacks nonlinearities differently or independently, for example, in [10] they propose an adaptive control to solve the problem of position regulation in a robot manipulator with uncertainty in the model and to demonstrate global asymptotic stability. Another control technique widely used to solve the uncertainty problem in the model and to reject external disturbances of the coupled type is the sliding mode control formulated by [11], where it achieves stability in finite time, but one of its drawbacks is the chattering effect on the control signal (high-frequency, low-amplitude oscillations). In [12] they solve the inconvenience of reaching the sliding surface and the effect of chattering is reduced with the proposal of the controller for integral sliding modes; in [13] they propose a control of variable structure composed of the control by integral sliding modes and a quadratic optimal linear controller for time-varying nonlinear systems in the presence of uncoupled perturbations. On the other hand, in [14] the super-twisting control applied to a manipulator robot that operates under Lipschitz uncertainty is proposed, the authors use the derivative of the Lyapunov function for a nominal model of the robot that is used for the design of the sliding surface, providing a theoretically exact convergence of uncertain system states to the origin using a continuous control signal, however, all of this excellent work is formulated in the continuous-time domain. In parallel, discrete-time domain controllers are proposed that address problems related to the performance of manipulator robots, for example, [15] proposes a discrete-time energy-based control with the IDA-PBC method for linear mechanical systems that use the midpoint discrete gradient that conserves energy, in [16] they employ an integration scheme to preserve the phase space of the flow of a discrete-time Hamiltonian system. Recently, in [17] they proposed discrete-time eigenvalue assignment based on the definition of an objective system, using the midpoint rule, the advantages compared to the continuous-time controller implementation include unconditional stability and setting the sampling period within the bounds of the sampling theorem. In [18] the formulation of a discrete-time energy-based control technique for mechanical systems, in particular manipulator robots, based on symplectic numerical integration is presented. Energy shaping is the central argument of passivity-based controls, and symplectic numerical integration schemes preserve a modified Hamiltonian. Therefore, according to Kotyczka and Toma, this method is appropriate to translate the energy formation step to discrete time.

The contribution of this work is the discretization of a non-linear discrete PD controller with compensation of the dynamics of the robot manipulator on the desired trajectory using the Euler numerical method, applied to a manipulator robot with three degrees of freedom called Niryo One.

The rest of the paper is organized as follows. In section 1, the Euler-Lagrange model for a robot manipulator. In section 2, the global non-linear PD controller for the robot manipulator is shown. Simulations made for 3-DOF industrial robots are presented. Finally, we give the conclusion.

2. DYNAMIC MODEL OF ROBOT MANIPULATOR AND USEFUL PROPERTIES

The Lagrange equations of motion for a robot manipulator on l DOF (degrees-of-freedom) with revolute joints and rigid links can be written as [5]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + f(\dot{q}) = \tau(t) + w(t), \quad (1)$$

where $q(t) \in \mathbb{R}^l$ is the vector of joint displacements, $\dot{q}(t) \in \mathbb{R}^l$ is the vector of joint velocities, $\tau(t) \in \mathbb{R}^l$ is the vector of applied torques, $t \in \mathbb{R}^l$, $M(q) \in \mathbb{R}^{lxl}$ is the symmetric positive definite inertial matrix, $C(q,\dot{q}) \in \mathbb{R}^{lxl}$ is the centrifugal and Coriolis forces matrix, $g(q) \in \mathbb{R}^l$ is the vector of gravitational torques, $w(t) \in \mathbb{R}^l$ is the matched disturbance vector i.e. uncertainty in the model, exogenous disturbances such as torques or forces due to possible physical contact with the robot body or interaction with external objects, and $f(\dot{q}) \in \mathbb{R}^l$ is the friction torques vector defined by

$$f(\dot{q}) = F\dot{q}, \quad (2)$$

here, the matrix $F \in \mathbb{R}^{lxl}$ is assumed diagonal and positive definite where the main diagonal includes the constant coefficients of viscous friction of each joint. All states are available for measurements for feedback.

Property 1. The inertial matrix $M(q)$ is symmetric and positive definite for all $q(t) \in \mathbb{R}^l$. The matrix $M^{-1}(q)$ exists and is positive definite as well, there exists a positive constant K_M such that for all $x, y, z \in \mathbb{R}^l$ we have [3]

$$\|M(x)z - M(y)z\| \leq K_M\|x-y\|\|z\| \quad (3)$$

Property 2. The centrifugal and Coriolis matrix $C(q,\dot{q})$ and the time derivative $\dot{M}(q)$ of the inertial matrix satisfy that skew-symmetry property as follows [3]

$$\dot{q}^T\left[\frac{1}{2}\dot{M}(q) - C(q,\dot{q})\right]\dot{q} = 0 \quad (4)$$

for all $q, \dot{q} \in \mathbb{R}^l$.

Property 3. There exists a positive constant K_{c1} such that for all $q, x, y \in \mathbb{R}^l$ we have [3]

$$\|C(q,x)y\| \leq K_{c1}\|x\|\|y\|. \quad (5)$$

Property 4. There exists a positive constant K_{c2} such that for all $s, v, x, y, z \in \mathbb{R}^l$ we have [3]

$$\|C(x,z)s - C(y,v)s\| \leq K_{c1}\|z-v\|\|s\| + K_{c2}\|x-y\|\|s\|\|z\|. \quad (6)$$

Property 5. There exists a positive constant K_g such that for all $x, y \in \mathbb{R}^l$ we have [3]

$$\|g(x) - g(y)\| \leq K_g\|x-y\|. \quad (7)$$

3. GLOBAL NON-LINEAR PD CONTROLLER FOR ROBOT MANIPULATOR

The global nonlinear PD motion problems can be established as follows: given a desired function ending vector position $q_d(t) \in \mathbb{R}^l$, the control objective for the tracking under study consists in making the systems globally asymptotically stable and ensuring that

$$\lim_{t\to\infty}\|q(t) - q_d(t)\| = 0, \quad (8)$$

for arbitrary initial condition $q(0) \in \mathbb{R}^l$ despite the presence of external disturbances.

Proposition 6. The global nonlinear PD controller of the form [3]

$$\tau(t) = M(q_d)\ddot{q}_d + C(q_d)\dot{q}_d + g(q_d) + f(\dot{q}_d) - K_p\tilde{q} - K_v\dot{\tilde{q}} \quad (9)$$

Where $\tilde{q} = q(t) - q_d(t)$, $\dot{\tilde{q}} = \dot{q} - \dot{q}_d$ are the position error vector and velocity error vector respectively, and K_p, K_v are (lxl) and symmetric positive definite matrix, imposes the manipulator motion around $q_d(t)$, when $w = 0$.

The controller to be constructed consists of the dynamics of the robot manipulator in the desired trajectory and friction torque compensators, a proportional-differential part that imposes the desired stability properties on the disturbance-free system motion.

The state-space representation of the close-loop system (1) and (9), in terms of the errors, is given by

$$\frac{d}{dt}\tilde{q} = \dot{\tilde{q}} \tag{10}$$
$$\frac{d}{dt}\dot{\tilde{q}} = M^{-1}(\tilde{q}+q_d)\left[\xi(t,\tilde{q},\dot{\tilde{q}}) + h(t,\tilde{q},\dot{\tilde{q}}) + w(t)\right],$$

where h is the residual dynamics, defined as:

$$h(t,\tilde{q},\dot{\tilde{q}}) = [M(q_d) - M(\tilde{q}+q_d)]\ddot{q}_d + \left[C(q_d,\dot{q}_d) - C(\tilde{q}+q_d,\dot{\tilde{q}}+\dot{q}_d)\right]\dot{q}_d + g(q_d) - g(\tilde{q}+q_d) + f(\dot{q}_d) - f(\dot{\tilde{q}}+\dot{q}_d), \tag{11}$$

and ξ is the PD controller and Coriolis matrix
$$\xi(t,\tilde{q},\dot{\tilde{q}}) = -K_p\tilde{q} - K_v\dot{\tilde{q}} - C(\tilde{q}+q_d,\dot{\tilde{q}}+\dot{q}_d)$$

Proposition 7. The inequality to demonstrate unicity of the equilibrium points| for the close-loop system (10) proposed by [3]

$$\lambda_{min}\{K_p\} > K_M\|\ddot{q}_d\| + K_{c2}\|\dot{q}_d\|^2 + K_g, \tag{12}$$

gives a sufficient condition for it (where λ_{min} is defined as the smaller eigenvalues). The inequality (12) is considered for the design of the global Non-linear PD Controller.

3.1. GLOBAL ASINTOTIC STABILITIES (GAS)

The global asymptotic stability is analyzed through the direct method of Lyapunov applied to close-loop system (10), the purpose is established conditions on the design matrices K_p and K_v that guarantee global asymptotic stability of the origin.

Proposition 8. Consider the Lyapunov candidate function [3] of the form

$$V(t,\tilde{q},\dot{\tilde{q}}) = \frac{1}{2}\dot{\tilde{q}}^T M(\tilde{q}+q_d)\dot{\tilde{q}} + \frac{1}{2}\tilde{q}^T K_p\tilde{q} + \gamma\tanh(\tilde{q})^T M(\tilde{q}+q_d)\dot{\tilde{q}}. \tag{13}$$

Proposition 9. We have that $V(t,0,0) = 0$ for $\tilde{q} = 0, \dot{\tilde{q}} = 0$ and $V(t,\tilde{q},\dot{\tilde{q}}) > 0$ for all $t \geq 0$ and $\tilde{q} \neq 0, \dot{\tilde{q}} \neq 0$ if the next inequality is hold [3]

$$\lambda_{min}\{K_p\} > \frac{\gamma^2\alpha_1^2\lambda_{max}^2\{M\}}{\lambda_{min}\{M\}}, \tag{14}$$

where γ, α are positives constants and λ_{max} is defined as the largest eigenvalues.

Proposition 10. The inequalities to demonstrate that the time derivative of the Lyapunov candidate function (13) is negative definite and therefore the equilibrium points for the close-loop system (10) are GAS if the next inequalities are holds

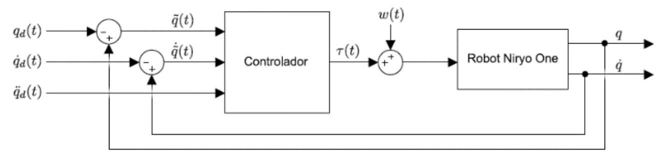
$$\lambda_{min}\{K_p\} > \lambda_{min}\{K_v\} + \alpha_3 K_{h2} \tag{15}$$

$$\lambda_{min}\{K_p\} > \frac{\alpha_3(0.5K_{h2} + 0.5K_{h2}\|\dot{q}_d\| + 0.5K_{h1})^2}{\frac{1}{\gamma}\lambda_{min}\{K_v\} - \alpha_4\lambda_{max}\{M\} + \frac{K_{h1}}{\gamma} + K_{c1}\alpha_2} + \lambda_{min}\{K_v\} + \alpha_3 K_{h2} \tag{16}$$

$$\lambda_{min}\{K_v\} > \gamma\left(\alpha_4\lambda_{max}\{M\} + \frac{K_{h1}}{\gamma} + K_{c1}\alpha_2\right) \tag{17}$$

The prepositions 7 to 10 are used for the design of the global Non-linear PD Controller for the robot manipulator in the continue-time domain (see Fig. 1).

Fig. 1. Closed-loop control system in the continuous-time domine.



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3.2. DİSCRETE NON-LİNEAR PD CONTROLLER

The discretization of the closed-loop controller system can be considered a good approximation if the sampling rates are high enough compared to the dynamics of the system [17], however, increasing the sampling time leads to performance degradation and instability [18]. Normally the control laws are derived with the implicit hypothesis of discretizing the continuous controllers, and derived from the complexity of the control algorithms, the use of a digital system is essential, usually, the control laws are discretized with an acceptable approximation to the continuous integration [1], this method is commonly used.

When the robots are controlled by digital processor, the input control signal is update at each sampling instant by a DAC (digital to analog convert) and a holding device so that the input is piecewise constant [1].

Assumption 11. We have that $\tau(t) \cong \tau(kT)$ where $kT \leq t \leq (k+1)T$, where T is the sampling time.

We used the backward and forward Euler methods applied to close-loop continuous system (10), and the following equation are obtained

$$\tilde{q}(t) \cong x_1(k-1) \tag{18}$$

$$\dot{\tilde{q}}(t) \cong \frac{x_1(k) - x_1(k-1)}{T} = x_2(k-1) \tag{19}$$

Using algebra and shift to right (k→k+1) in the equations (18) and (19), we have the derivative in differences:

$$x_1(k+1) = x_1(k) + Tx_2(k) \tag{20}$$
$$x_2(k+1) = x_2(k) + TM^{-1}(x_1 + q_d)$$
$$\left[-C\left(x_1 + q_d, x_2 + \frac{q_d(k) - q_d(k-1)}{T}\right)x_2 + K_p x_1 + K_v x_2 + h(k, x_1, x_2) - w(k)\right]$$

4. RESULT

The simulation setup involves a robot manipulator (see Fig. 2). The base of the mechanical robot has a horizontal revolute joint q_1, whereas two link have vertical revolute joints q_2 and q_3. The remaining DOF corresponds to the end effector orientation. Nominal parameters of mechanical manipulator are summarized in Table I. The robot manipulator was required to move in the space from the initial conditions $q(0) = \left[\frac{\pi}{2} \quad \frac{-\pi}{2} \quad \frac{\pi}{2}\right]^T$(reference for each joint are shown in Fig. 2) to the desired trajectories

$$q_d(k) = \left[\frac{\pi}{4}\left(1 - e^{-2(kT)^2}\right) + \frac{\pi}{4}\left(1 - e^{-2(kT)^3}\right)sen(0.2\pi kT)\frac{\pi}{12}\left(1 - e^{-2(kT)^3}\right) + \frac{\pi}{12}\left(1 - e^{-2(kT)^3}\right)sen(0.4\pi kT)\frac{\pi}{20}\left(1 - e^{-2(kT)^3}\right) + \frac{\pi}{20}\left(1 - e^{-2(kT)^3}\right)sen(0.6\pi kT)\right]. \tag{21}$$

The initial velocity $\dot{q}(0) \in \mathbb{R}^3$ were zeros for the simulation. We achieved the control goal by implementing the discrete and non-linear PD controller, with the follow parameters

$$K_p = diag\{632.8 \quad 631.8 \quad 630.8\} \tag{22}$$

$$K_v = diag\{4.49 \quad 4.39 \quad 4.29\} \tag{23}$$

The simulation was performed affecting the model with harmonic functions as external disturbance, that is,

$$w_i(k) = \frac{\pi}{8}\text{sen}\,(2\pi ikT) \tag{24}$$

Table 1. Parameter of the mechanical manipulator

Description	Notation	Values	Units
Length of link 1	l_1	0.16048	m
Length of link 2	l_2	0.297	m
Mass of link 1	m_1	1	kg
Mass of link 2	m_2	1.631	kg
Inertial 1	I_1	$4.676x10^{-3}$	kgm^2
Inertial 2	I_2	$1.468x10^{-3}$	kgm^2
Inertial 3	I_3	$7.468x10^{-3}$	kgm^2
Friction coeff. 1	α_{11}	0.01	Nm
Friction coeff. 2	α_{22}	0.02	Nm
Friction coeff. 3	α_{33}	0.015	Nm
Gravity acceleration	g	9.8	m/s

Source: self made.

Fig. 2. Manipulator robot Niryo One.

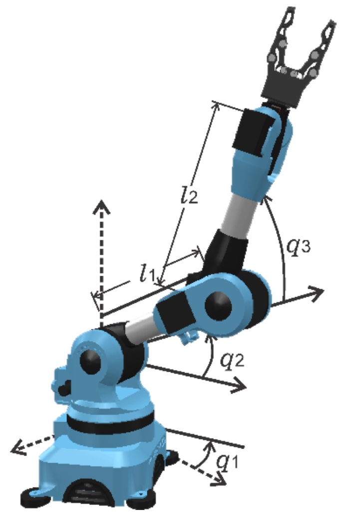

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Figure 3, which illustrate angular position and desired angular position, corroborates disturbance attenuation of the closed-loop system. In figure 4 is illustrated the phase plane of the close-loop system without disturbance, corroborates that origin is global asyntotic stabilitiy.

Fig. 3. Performance for the closed-loop system.

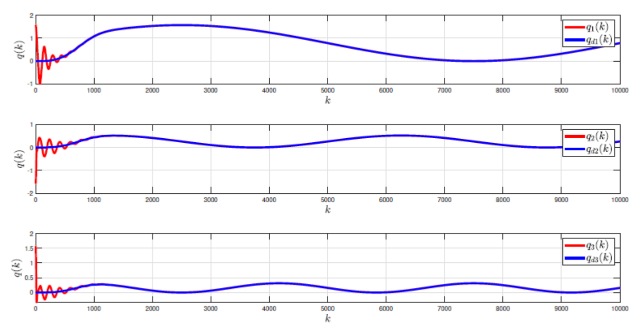

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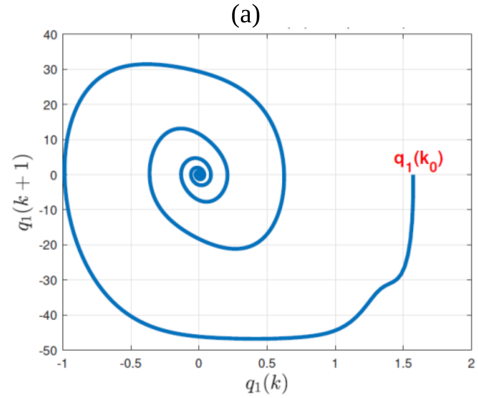
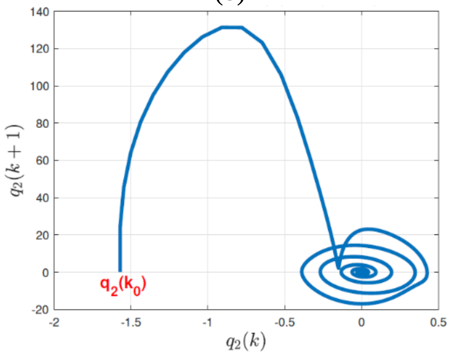
Fig. 4. Phase plane for close-loop systems, (a) phase plane of $q_1(k), q_1(k+1)$, (b) phase plane of $q_2(k), q_2(k+1)$, (c) phase plane of $q_3(k), q_3(k+1)$.

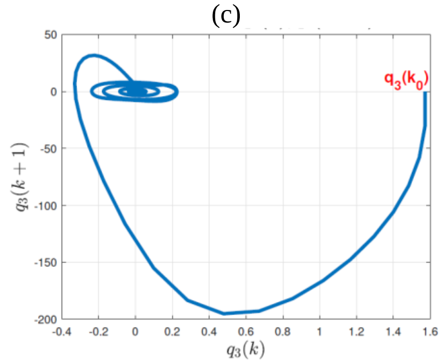
(a)



(b)



(c)



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5. CONCLUSION

We presented a global solution of the discrete and non-linear PD motion problem applied to the Niryo One manipulator robot and validate in simulation that global asymptotic stability is hold in both time domine. The objective of control is achieved in the presence of a disturbance vector.

APPENDIX

The equation of motion of the simulation manipulator governed by (1) was specified by applying the Euler-Lagrange formulation [3],

$$M(q) = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{23} & m_{33} \end{bmatrix},$$

where

$$m_{11} = m_1 l_1^2 cos^2(q_2) + I_1 + m_2[l_1 \cos(q_2) + l_2 \cos(q_2 + q_3)]$$
$$m_{22} = m_1 l_1^2 + m_2 l_1^2 + m_2 l_2^2 + 2 m_2 l_1 l_2 \cos(q_3) + I_2 + I_3$$
$$m_{23} = m_2 l_2^2 + m_2 l_1 l_2 \cos(q_3) + I_3$$
$$m_{33} = m_2 l_2^2 + I_3$$

$$C(q,\dot{q}) = \begin{bmatrix} c_{11} & c_{12} & 0 \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & 0 \end{bmatrix},$$

$$c_{11}(q,\dot{q}) = -2m_2[l_1 \cos(q_2) + l_2 \cos(q_2 + q_3)][l_1 \operatorname{sen}(q_2)\,\dot{q}_2 + l_2 \operatorname{sen}(q_2 + q_3)\,(\dot{q}_2 + \dot{q}_3)]$$

$$c_{12}(q,\dot{q}) = -2m_1 l_1 \operatorname{sen}(q_2) \cos(q_2)\,\dot{q}_1$$

$$c_{21}(q,\dot{q}) = m_2 l_1^2 \operatorname{sen}(q_2) \cos(q_2)\,\dot{q}_1 - m_2[l_1 \cos(q_2) + l_2 \cos(q_2 + q_3)][-l_1 \operatorname{sen}(q_2) - l_2 \operatorname{sen}(q_2 + q_3)]\dot{q}_1$$
$$c_{22}(q,\dot{q}) = -2m_2 l_1\, l_2 \operatorname{sen}(q_3)\,\dot{q}_3$$
$$c_{23}(q,\dot{q}) = -m_2 l_1\, l_2 \operatorname{sen}(q_3)\,\dot{q}_3$$
$$c_{31}(q,\dot{q}) = m_2[l_1 \cos(q_2) + l_2 \cos(q_2 + q_3)][l_2 \operatorname{sen}(q_2 + q_3)]\dot{q}_1$$
$$c_{32}(q,\dot{q}) = m_2 l_1\, l_2 \operatorname{sen}(q_3)\,(\dot{q}_2 + \dot{q}_3)$$

$$\begin{bmatrix} 0 \\ g_2(q) \\ g_3(q) \end{bmatrix},$$

$$g_2(q) = [m_1 l_1 \cos(q_2) + m_2 l_1 \cos(q_2) + m_2 l_2 \cos(q_2 + q_3)]g$$
$$g_3(q) = [m_2 l_2 \cos(q_2 + q_3)]g$$

REFEENCES

[1] S. Nicosia, P. Tomei, and A. Tornambè, “Discrete-time modeling and control of robotic manipulators”, Journal of Intelligent and Robotic Systems, vol. 2, pp. 411–423, (1989).

[2] S. Arimoto, “Control theory of nonlinear mechanical systems. A Passivity-based and Circuit-theoretic Approach”, 1996.

[3] R. Kelly, V. S. Davila, and J.A.L. Perez, “Control of robot manipulators in joint space”, Springer Science & Business Media, 2005.

[4] M. Spong, K. Khorasani, and P. Kokotovic, “An integral manifold approach to the feedback control of flexible joint robots”. IEEE Journal on Robotics and Automation, vol. 3, pp. 291–300, 1987.

[5] M. Spong, and M. Vidyasagar, “Robot dynamics and control”. John Wiley & Sons, 2008.

[6] R. Kelly, and C. Monroy, “On robustness of pd control with gravity compensation of torque–driven robot manipulators”. In 2019 International Conference on Control, Automation and Diagnosis (ICCAD), pp. 1–5, 2019.

[7] Y. Orlov, and L. Aguilar, “Advanced H control: Towards nonsmooth theory and applications”, Springer Science & Business Media, 2014.

[8] V. Utkin, J. Guldner, and J. Shi, “Sliding mode control in electro-mechanical systems”, CRC press, 2017.

[9] H. Sage, M. De Mathelin, and E. Ostertag, “Robust control of robot manipula- tors: a survey”. International Journal of control, vol. 72, pp.1498–1522, 1999.

[10] Y. Dai, S. Shi, and N. Zheng, “Adaptive control of robot manipulators with only position measurements”. IFAC Proceedings Vol. 32, pp. 665–670, 1999.

[11] V. I. Utkin, “Sliding modes in control and optimization”. Springer Science & Business Media, 2013.

[12] W. Cao, and J. Xu, “Nonlinear integral-type sliding surface for both matched and unmatched uncertain systems”. IEEE Transactions on Automatic Control, vol. 49, pp. 1355–1360, 2004.

[13] M. Estrada, L. Fridman, and J. Moreno, “Control of fully actuated mechanical systems via super-twisting based

Lyapunov redesign". IFAC-PapersOnLine, vol. 53, pp. 5117–5121, 2020.
[14] S. Aoues, D. Matignon, and D. Alazard, "Control of a flexible spacecraft using discrete ida-pbc design" IFAC-PapersOnLine, vol. 48, pp. 188–193, 2015.
[15] E. Hairer, G. Wanner, and C. Lubich, "Symplectic integration of Hamiltonian systems. In Geometric Numerical Integration", Springer, 2006.
[16] P. Kotyczka, and L. Lefèvre, "Discrete-time control design based on symplectic integration: Linear systems", IFAC-PapersOnLine, vol. 53, pp.7563–7568, 2020.
[17] P. Kotyczka, and T. Thoma, "Symplectic discrete-time energy-based control for nonlinear mechanical systems", Automatica, vol. 133, pp.1-13, 2021.
[18] H. Chen, and P.H. Rossi, "The theory-driven approach to validity", Evaluation and program planning, vol. 10, pp. 95–103, 1987, doi: 10.1016/0149-7189(87)90025-5