

## Design and implementation of a fuzzy LQR applied to the altitude control of an Innosat.

Ing. Juan Felipe Tabares-Hernández<sup>a</sup>, Dr. Juan Fernando García-Mejía<sup>b</sup>, Dr. Allan Antonio Flores-Fuentes<sup>c</sup>, Dr. Everardo Efrén Granda-Gutiérrez<sup>d</sup>, M. en I. Ricardo Arturo Vázquez-Robledo<sup>e</sup>, M. en I. Rigoberto Reyes-Morales<sup>f</sup>

<sup>a</sup> Universidad Autónoma del Estado de México, Centro Universitario UAEMex Atlacomulco [jtabaresh001@alumno.uaemex.mx](mailto:jtabaresh001@alumno.uaemex.mx)

<sup>b</sup> Universidad Autónoma del Estado de México, Centro Universitario UAEMex Atlacomulco [fgarciam@uaemex.mx](mailto:fgarciam@uaemex.mx)

<sup>c</sup> Universidad Autónoma del Estado de México, Centro Universitario UAEMex Atlacomulco [aafloresf@uaemex.mx](mailto:aafloresf@uaemex.mx)

<sup>d</sup> Universidad Autónoma del Estado de México, Centro Universitario UAEMex Atlacomulco [ecgrandag@uaemex.mx](mailto:ecgrandag@uaemex.mx)

<sup>e</sup> Centro Regional de Desarrollo Espacial Atlacomulco [vazquez.ricardo@aem.mx](mailto:vazquez.ricardo@aem.mx)

<sup>f</sup> Centro Regional de Desarrollo Espacial Atlacomulco [reyes.rigoberto@aem.mx](mailto:reyes.rigoberto@aem.mx)

### Resumen

El control automático es un área de la ingeniería que se ha visto beneficiada gracias a la inteligencia artificial, permitiendo el desarrollo de sistemas autorregulables aplicados a diferentes áreas de la ciencia; un ejemplo de ello son los sistemas de regulación de altitud de los satélites pequeños o nanosatélites, permitiéndoles modificar su orientación en órbita para llevar a cabo su misión con éxito. En este trabajo se utilizan técnicas de inteligencia artificial (lógica difusa) y de control óptimo (Reguladores lineales cuadráticos o LQR) para regular el comportamiento de 2 ejes de rotación de un nanosatélite Innosat. Los resultados son comparados con el rendimiento obtenido por un LQR con ajuste clásico y con un LQR con ajuste genético.

**Palabras clave**— Control de altitud, InnoSat, Lógica difusa, Reguladores Lineales cuadráticos.

### Abstract

*Automatic control is an area of engineering that has benefited from artificial intelligence, allowing the development of self-regulating systems applied to different areas of science; an example of this are the altitude regulation systems of small satellites or nanosatellites, allowing them to modify their orientation in orbit to carry out their mission successfully. In this work, artificial intelligence (fuzzy logic) and optimal control techniques (Linear Quadratic Regulators or LQR) are used to regulate the 2-axis rotation behavior of an Innosat nanosatellite. The results are compared with the performance obtained by a classically tuned LQR and a genetically tuned LQR.*

**Keywords**— Altitude Controller, Fuzzy Logic, InnoSAT Quadratic Linear Regulator.

## 1. INTRODUCTION

Satellites are widely used devices in the telecommunications industry allowing the sending and receiving of information

over long distances, however, their large size and high cost created an area of opportunity for the development of small satellites or nano satellites similar in performance to their regular sized counterparts, but with reduced cost and weight. [1].

Due to their size, nanosatellites are more susceptible to external disturbances, such as solar wind, gravity or radiation, which is why they require an altitude control system that allows them to maintain their position and regulate it when it is modified by any external factor [2].

An example of a nanosatellite currently used in research missions is the Innovative Satellite, also known as InnoSat, a cubesat-type nanosatellite developed by the Malaysian space agency, which has an altitude control system that allows it to regulate its position in orbit by means of reaction wheels [3].

In the state of the art, we can find multiple works that have used different control techniques to regulate the altitude of this nanosatellite, some of them focused on classical control (PID, PI, PD) [4], others focused on modern control (LQR) [5]. and others focused on intelligent control (Fuzzy Logic, Genetic Algorithms) [3] [6][7].

It is possible to highlight that techniques based on classical control alone are insufficient to regulate the InnoSat's altitude, due to its unstable nature. On the other hand, techniques based on intelligent and modern control have shown satisfactory results for the stabilization and control of the system. [4].

Due to the above, in this work a fuzzy logic based LQR (Linear Quadratic Regulator) was developed that allows the stabilization and control of the altitude system of an InnoSat, comparing its performance with that obtained by a classically adjusted LQR and with that obtained by an LQR adjusted by means of evolutionary algorithms.

## 1. THEORETICAL FRAMEWORK

This section shows the theoretical information that supports the proposal documented in this research process, it is composed by the foundation of the LQR, and the fuzzy logic used, in addition to the dynamics represented by the InnoSat transfer function.

### 2.1 Linear Quadratic Regulator

It is a technique of optimal control based on the analysis of state variables and control inputs, having as main objective the minimization of a cost function, for which it is necessary to represent the transfer function of the system in the form of a vector of state variables, this is possible by means of the system of equations (1)[8]:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \dots\dots\dots(1)$$

Where  $A \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^{n \times m}$ ,  $C \in \mathbb{R}^{p \times n}$ , and represent the inputs and outputs of the matrices, on the other hand, to compute the cost function  $J$  to be minimized we employ Eq. (2) [8]:

$$J = \int_0^{\infty} [x^T(t)Qx(t) + Ru^T(t)]dt \dots\dots\dots(2)$$

Where  $Q \in \mathbb{R}^{n \times n}$  y  $R \in \mathbb{R}^{m \times m}$ , and are positive constant matrices.

Therefore, it is possible to calculate the control law based on Eq. (3) [8]:

$$u(t) = -Kx(t) \dots\dots\dots(3)$$

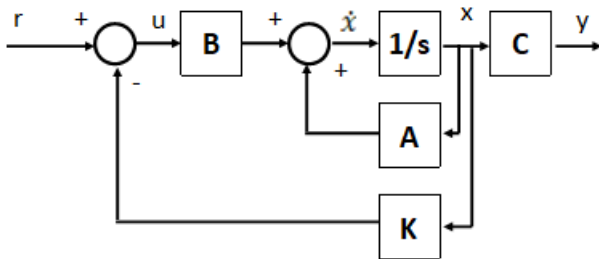
Where  $K$  is the optimal profit matrix for the system and is determined by Eq. (4) [8].

$$K = R^{-1}B^T P \dots\dots\dots(4)$$

Where  $P$  is the single positive solution defined from the Riccati equation  $A^T P + AP - PBR^{-1}B^T P + Q = 0$  [8].

Based on the above, it is possible to represent the operation of an LQR by means of Fig. 1 [9].

Figure 1 LQR control scheme.



Source: Own elaboration based on [9].

## 2.2 Fuzzy Logic

Fuzzy logic is an intelligent control technique that allows modeling imprecise behaviors to reach reasoned conclusions, since, unlike classical logic that has only two states, it allows a gradual progression from set membership to non-membership.

A fuzzy set of values is represented by Eq. (5) [10]:

$$A = \{(X, \mu_A(X))\} | x \in X \dots\dots\dots(5)$$

Where  $A$  is a fuzzy set in  $X$ ,  $X$  is a data set or universe of discourse,  $\mu_A(X)$  is the membership function of  $X$  in  $A$ .

If the universe of discourse is finite or discrete it is possible to represent it as the summation of several membership functions, on the other hand, if  $X$  is continuous and infinite it

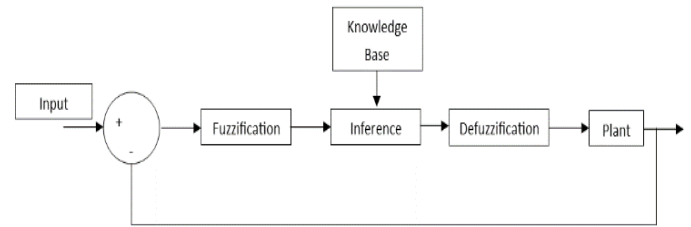
is expressed in terms of an integral, both cases are represented in Eq. (6) and Eq. (7) [10] respectively.

$$A = \left\{ \frac{\mu_A(X_1)}{X_1} + \frac{\mu_A(X_2)}{X_2} + \dots \right\} = \sum_i \frac{\mu_A(X_i)}{X_i} \dots\dots\dots(6)$$

$$A = \left\{ \int \frac{\mu_A(X)}{X} \right\} \dots\dots\dots(7)$$

Fig. 2 [11] depicts the operation of a fuzzy system, which is composed of three stages:

Figure 2 Schematic diagram of a fuzzy controller.



Source: Own elaboration.

### 2.1.1 Fuzzification

This stage is constructed by the membership functions of the system, which are responsible for mapping the universe of discourse and the set of real numbers to determine the degree to which a linguistic value is assigned to a variable  $X$ .

Eq. (8) and Eq. (9) [12] represent the behavior of triangular and trapezoidal type membership functions respectively used due to the low consumption of computational resources required for their implementation.

$$f(x; a, b, c) = \begin{cases} 0, & x < a \\ \frac{b-a}{c-x}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & x > c \end{cases} \dots\dots\dots(8)$$

$$f(x; a, b, c, d) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{d-x}{d-c}, & c \leq x \leq d \\ 0, & d < x \end{cases} \dots\dots\dots(9)$$

Where  $a, b, c$  and  $d$  are constants.

### 2.1.2 Knowledge base or Inference Engine

Inference in fuzzy systems is performed by means of the compositional rule of inference, and can be represented by the following expression:

If "x" is "A" then "y" is "B".

It is possible to implement it by means of a max-min composition as shown in Eq. (10) [11].

$$\mu_A(a_1, \dots, a_n) \wedge \mu_A(b_1, \dots, b_n) \wedge \max \left\{ \min \left\{ \mu_A(a_i), \mu_A(b_j) \right\} \right\} \dots \dots \dots (10)$$

Where  $a_i$  and  $b_j$  are linguistic labels,  $\mu_A(a_i)$  y  $\mu_A(b_j) \in [0,1], i = 1, \dots, n, j = 1, \dots, m, n \geq 1$  y  $m \leq \infty$ .

**2.1.3 Defuzzification**

In this stage, the fuzzy output obtained from the inference is taken and a transformation process is applied in terms of the discourse domain of the problem to be treated. One of the most widely used methods for defuzzification is the centroid method represented in Eq. (11) [11].

$$Z^* = \frac{\int \mu_A(z) * z dz}{\int \mu_A(z) dz} \dots \dots \dots (11)$$

**2.3 InnoSat Dynamics**

The denomination of nanosatellite is assigned to any artificial device that orbits the earth and has a weight greater than or equal to 1kg and less than 10kg, these are deployed in low orbit and perform monitoring or guidance missions of no more than 1 year in duration [13].

The cube type structure is the most used in this type of satellites since it allows modularizing its components, adding new payloads or replacing inefficient subsystems, within this denomination we find InnoSat [13].

Developed in 2010 by the Malaysian space agency in conjunction with Universiti Sains Malaysia (USM), Universiti Teknologi Malaysia (UTM) and Universiti Malaysia Perlis (UniMAP), its main objective was to serve as a tool to test various altitude systems, later used for simple satellite monitoring and positioning missions [13]. In 2018, an upgraded version of the nanosatellite called InnoSat 2 was put into orbit, with an upgrade to its monitoring systems [14].

Eq. (12) and Eq. (13) [4] represent the behavior of the InnoSat altitude system in its Yaw and Pitch rotation axes.

$$\phi(s) = \frac{s^2 + 0.3051s + 0.2040}{s^4 + 1.1050s^2 + 0.1650} \dots \dots \dots (12)$$

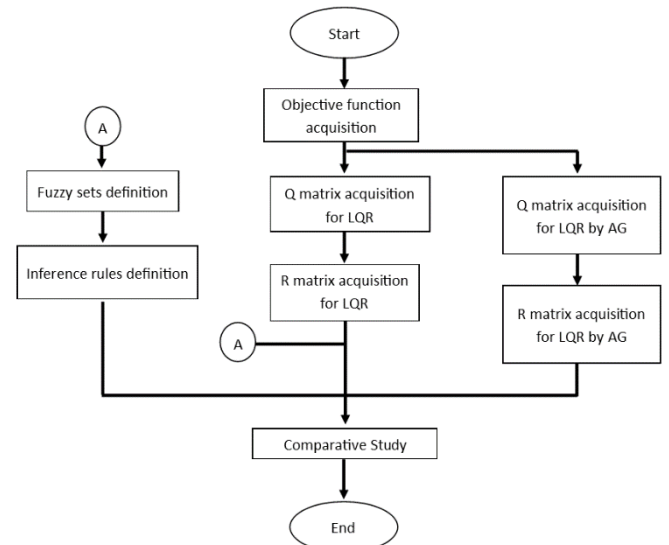
$$\theta(s) = \frac{1}{s^2 - 7.1138 \times 10^{-3}} \dots \dots \dots (13)$$

The transfer functions must be converted to their state matrix form, and from them calculate their gains Q and R; this process is discussed in the next section.

**3. METHODOLOGY**

The methodology used in the present work is shown in Fig. 3, where a fuzzy logic based LQR is compared with the results obtained by a classical LQR and a genetic LQR, whose development can be seen in detail in [15].

Figure 3 Methodological scheme.



Source: Own elaboration.

A Mamdani fuzzy controller (MFC) of SISO type (single input, single output) was developed, because it allows to express knowledge with simplicity in the inference stage. Table 1 shows the fuzzy sets associated to the input variable error (with a range from -20 to 100 %), Table 2 shows the output variable with its fuzzy sets (with a range from -2.5 to 20) and finally Table 3 shows the table of inference rules.

Table 1 Input variable with its fuzzy sets

Variable	Fuzzy sets	Set size for $\phi$	Set size for $\theta$
error (%)	ne Negative error	-20, -15, 0.0003077	-20, -15, 0.0003077
	ze Zero error	-5, -1.245, 0.7554, 5	-5, -1.245, 0.7554, 5
	pe Positive error	0.0003077, 18.47, 100	0.0003077, 18.47, 100

Table 2 Output variable with its fuzzy sets

Variable	Fuzzy sets	Set size for $\phi$	Set size for $\theta$
Control slope (Cs)	lo Low output	-2.5, -1.563, 6.5	-2.5, 4.101, 9.661
	mo Medium output	6.5, 7.813, 9.687, 11	9.592, 10.9, 12.78, 14.09
	ho High output	11, 13.25, 20	14.09, 19.34, 20.0

Table 3 Inference rules

No.	Rules
1	If <i>error</i> is (ne) then <i>Cs</i> is (lo)
2	If <i>error</i> is (ze) then <i>Cs</i> is (mo)
3	If <i>error</i> is (pe) then <i>Cs</i> is (ho)

The state matrices, generated from the InnoSat transfer functions, are shown in Table 4, while the Q and R matrices are shown in Table 5.

Table 4 State matrices

Matrices Angles	A	B	C	D
$\phi$	$\begin{bmatrix} 0 & -1.105 & 0 & -0.1650 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$[0 \ 1 \ 0.3051 \ 0.2040]$	$[0]$
$\theta$	$\begin{bmatrix} 0 & 0.0071 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$[0 \ 1]$	$[0]$

Source: Own elaboration.

Table 5 Q and R matrices

Matrices Angles	Q	R
$\phi$	$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0.3051 & 0.2040 \\ 0 & 0.3051 & 0.0931 & 0.0622 \\ 0 & 0.2040 & 0.0622 & 0.0416 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$
$\theta$	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Source: Own elaboration.

### 3. RESULTS

Table 6 shows the numerical results obtained for the LQR with classical adjustment, the LQR with genetic adjustment and the LQR with fuzzy control, establishing as comparison parameters the settling time ( $t_s$ ), the overshoot value ( $M_p$ ) and the root mean square error value ( $e_{rms}$ ).

The results obtained from the fuzzy LQR controller show a shorter settling time compared to LQR with classical adjustment and genetic LQR, however, this technique presents a higher overshoot, as well as a higher RMS error compared to the previous techniques.

Fig. 4 and Fig. 5 show the comparative responses obtained from the closed-loop control for the Yaw ( $\phi$ ) and Pitch ( $\theta$ ) axes. In the first case it is possible to observe that the fuzzy LQR can follow the reference better than the classically tuned controller, and slightly worse than the genetically tuned controller, on the other hand, in the case of the Pitch ( $\theta$ ) axis, it can be observed that the classically and genetically tuned controllers reached the reference while the fuzzy controller exceeded it.

Table 6 Performance criteria comparison.

Parameter	Performance criteria analytical method			Performance criteria with Genetic Algorithm Adjusted Matrices			Performance criteria with analytical method and fuzzy controller		
	$t_s(s)$	$M_p$	$e_{rms}$	$t_s(s)$	$M_p$	$e_{rms}$	$t_s(s)$	$M_p$	$e_{rms}$
$\phi$	14	0.9933	0.2533	14	1.0490	0.1412	5	1.92	1.0171
$\theta$	10	1.0422	0.2533	10	1.0422	0.2533	8	1.89	0.9989

Figure 4 Yaw ( $\phi$ ) axis control response

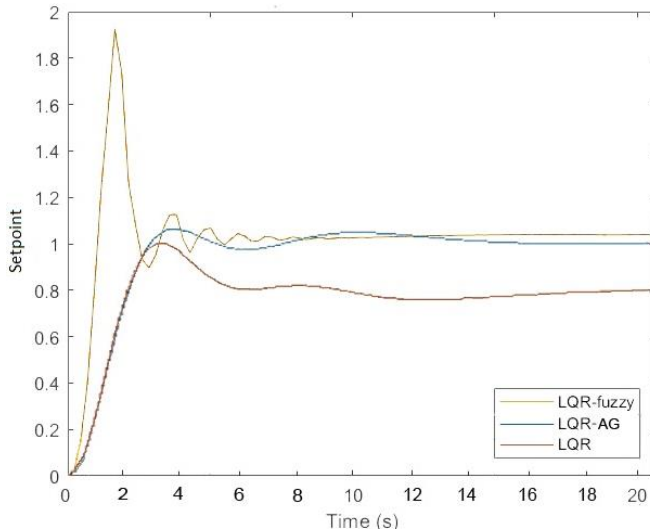
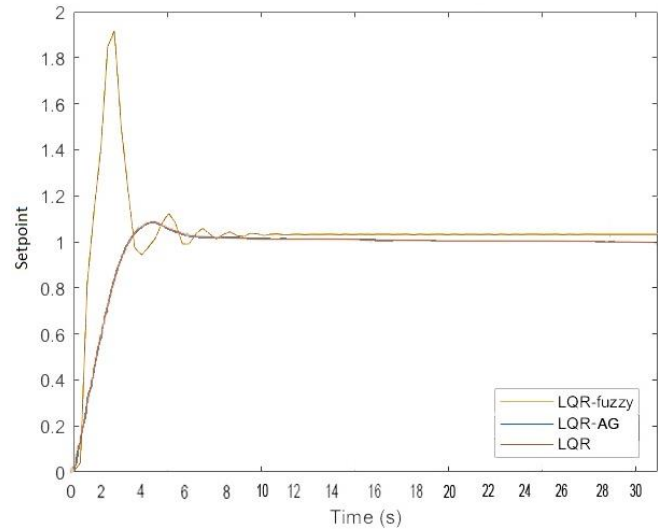


Figure 5 Pitch ( $\theta$ ) axis control response



#### 4. CONCLUSIONS Y RECOMENDACIONES

After the analysis of the results, it is possible to conclude that the use of fuzzy logic with classical adjustment in conjunction with an LQR can regulate the position of an Innosat. However, its performance is inferior to that presented by an LQR with genetic adjustment, and although, it is able to reach the reference better than the LQR with classical adjustment in the Yaw axis ( $\phi$ ), this is not the case in the Roll axis ( $\theta$ ), due to this and to the fact that its overshoot and erms value are relatively high, this technique is discarded as a feasible option for implementation in a real environment.

Therefore, as future work, it is proposed to continue with the use of evolutionary algorithms as a basis for the development of new controllers and to compare their efficiency with the results obtained so far.

#### 5. REFERENCES

[1] A. Camps, "Nanosatellites and Applications to Commercial and Scientific Missions", *Satellites Missions and Technologies for Geosciences*. IntechOpen, Jul. 22, 2020. doi: 10.5772/intechopen.90039.

[2] Fariñas-Álvarez, Noelia; Navarro-Medina, Fermín; González-Jorge, Higinio, Evaluación metrológica de instalación tipo helmholtz para ensayar los sistemas de determinación y control de actitud (ADCS) de pequeños satélites. *Revista DYNA*, ISSN 0012-7361, ISSN-e 0012-7361, Vol. 97, N° 3, 2022.

[3] Adaptive fuzzy logic controller with direct action type structures for InnoSAT attitude control system, F. Bakri, M. Mashor, S. Mashor, IOP Conference Series: Materials Science and Engineering, 152, 2016, DOI: 10.1088/1757-899X/152/1/012032

[4] Performance of Manual and Auto-Tuning PID Controller for Unstable Plant - Nano Satellite Attitude Control System, H. Hani, M. Hani, The 6th International Conference on Cyber

and IT Service Management, 2018, DOI: 10.1109/CITSM.2018.8674375

[5] Hashim, F., Mashor, M. Y., & Sharun, S. M. (2015). Full-And Reduced-Order Compensator for Innosat Attitude Control. *Jurnal Teknologi*, 76(12). <https://doi.org/10.11113/jt.v76.5883>.

[6] R. Rakesh et al., "Dynamics and control system design of a polar low-earth orbit nanosatellite 'Parikshit,'" 2015 International Conference on Advances in Computing, Communications and Informatics (ICACCI), 2015, pp. 607-613, doi: 10.1109/ICACCI.2015.7275676.

[7] Bakri, F & Mashor, Mohd & Sharun, Siti & Sarpinah, Bibi & Abu Bakar, Zahari. (2016). Adaptive fuzzy logic controller with direct action type structures for InnoSAT attitude control system. IOP Conference Series: Materials Science and Engineering. 152. 012032. 10.1088/1757-899X/152/1/012032.

[8] Aisha Sir Elkhateem, Seref Naci Engin. Robust LQR and LQR-PI control strategies based on adaptive weighting matrix selection for a UAV position and attitude tracking control. *Alexandria Engineering Journal*. Volume 61, Issue 8.2022. Pages 6275-6292. ISSN 1110-0168. <https://doi.org/10.1016/j.aej.2021.11.057>.

[9] Maraslidis, G.S.; Kottas, T.L.; Tsipouras, M.G.; Fragulis, G.F. Design of a Fuzzy Logic Controller for the Double Pendulum Inverted on a Cart. *Information* 2022, 13, 379. <https://doi.org/10.3390/info13080379>.

[10] Pérez, Iván, León, Betzabeth. *Lógica difusa para principiantes: teoría y práctica*. Caracas: Universidad Valle del Momboy, 2007.

[11] Iler, W., y Buckley, J. *Fuzzy Expert Systems and Fuzzy Reasoning*: John Wiley & Sons; 2005.

[12] Chi Z, Yan H, Pham T. *Fuzzy Algorithms: With Applications to Image Processing and Pattern Recognition*. World Scientific, 1996.

[13] M.Fadly, Md Azlin Md Said. Development of Attitude Determination for Student Pico-Satellite INNOSAT. Regional Conference on Mechanical and Aerospace Technology. Bali, 2010.

[14] Krebs, Gunter D. "*InnoSat 2*". Gunter's Space Page. Retrieved February 03, 2024, from [https://space.skyrocket.de/doc\\_sdat/innosat-2.htm](https://space.skyrocket.de/doc_sdat/innosat-2.htm).

[15] García-Mejía, J. F., Tabares Hernández, J. F., Flores Fuentes, A. A., Granda Gutiérrez, E. E., Ajuste de reguladores lineales cuadráticos por medio de algoritmos evolutivos aplicados a un Innosat, Número Especial de la Revista Aristas Ciencia Básica y Aplicada., Vol. 10 Núm. 18 (2023).