White Shark Optimizer for LQR tuning applied to position control of a 2-DOF helicopter

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Abstract

The design of optimal controllers for complex dynamic systems is a key application area of Artificial Intelligence. This work proposes the optimization of a Linear Quadratic Regulator (LQR) applied to the control of a 2 degree-of-freedom (2-DOF) helicopter using the bio-inspired White Shark Optimizer (WSO) algorithm. Conventional LQR parameters tuning is typically performed using analytical methods, which may be inefficient given the complexity and nonlinearity inherent in these systems. As an alternative, WSO is implemented to adjust these parameters and is compared against an LQR designed using the traditional analytical method and another optimized with Particle Swarm Optimization (PSO). The evaluation is conducted through simulations and an inferential statistical analysis of both heuristics. The results show that WSO enables a more efficient LQR tuning in terms of performance indices, including the Integral Absolute Error (IAE), the Integral Squared Error (ISE), and their time-weighted variants (ITAE and ITSE), compared to other tuning approaches.

Keywords — Optimization, Linear Quadratic Regulator, White Shark Optimizer, 2-DOF Helicopter.

Resumen

El diseño de controladores óptimos para sistemas dinámicos complejos es un área de aplicación de la Inteligencia Artificial. En este trabajo, se propone la optimización de un Regulador Cuadrático Lineal (LQR) aplicado al control de un helicóptero de dos grados de libertad (2-DOF) mediante el algoritmo bioinspirado White Shark Optimizer (WSO). La sintonización convencional de los parámetros del LQR se lleva a cabo mediante métodos analíticos, los cuales pueden resultar ineficientes ante la complejidad y no linealidad inherente de estos sistemas. Como alternativa, se implementa el WSO para ajustar de estos parámetros, y se compara con un LQR diseñado mediante el método analítico tradicional y otro optimizado con Particle Swarm Optimization (PSO). La evaluación se realiza a través de simulaciones y un análisis estadístico inferencial de ambas heurísticas. Los resultados muestran que el WSO permite una sintonización más eficiente del LQR en términos de los índices de desempeño, incluyendo el Error Absoluto Integral (IAE), el Error Cuadrático Integral (ISE) y sus versiones ponderadas en el tiempo (ITAE e ITSE), respecto de los otros enfoques de sintonización.

***Palabras clave** — Optimización, Regulador Cuadrático Lineal, White Shark Optimizer, Helicóptero de 2-DOF.*

1. INTRODUCTION

One area of Artificial Intelligence is the development of heuristic algorithms to solve complex optimization problems. Inspired by human, evolutionary, biological or physical processes, these algorithms allow to address problems where the objective function is to be maximized or minimized [1]. One of their applications is the tuning of controllers, where the dynamics and limitations of the system to be controlled make traditional methods inefficient.

A widely used case study in control is the 2-degree-of-freedom (2-DOF) helicopter, a platform that allows to represent simplified models of more complex aerospace systems [2]. Its structure and dynamics make it an ideal system to evaluate control and optimization strategies. Various approaches have been proposed to regulate its behavior, highlighting methodologies such as robust control [3][4], intelligent control [5][6] and optimal control [7][8].

In the field of optimal control, the Linear Quadratic Regulator (LQR) is one of the most used strategies due to its ability to stabilize dynamic systems [7]. However, its effectiveness depends on the proper selection of the weighting matrices Q and R. This adjustment, often performed empirically or by deterministic analytical methods, can lead to suboptimal or inefficient solutions, commonly associated with the complexity and non-linearity of the systems.

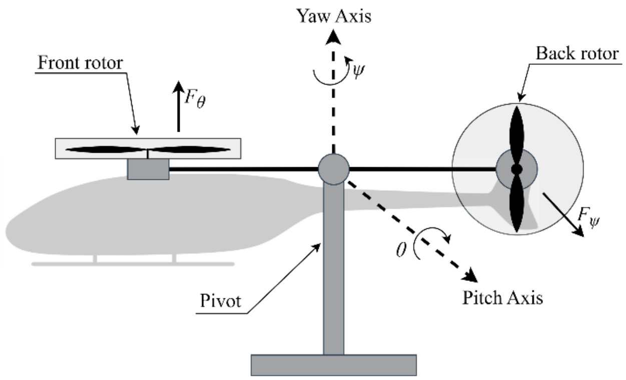
In the literature, various alternatives based on heuristic algorithms have been documented to tune LQR controllers [9][10]. As an example of this type of algorithm, the use of the White Shark Optimizer (WSO) is proposed, which emulates the hunting patterns of white sharks, allowing the exploration of wide search spaces to identify optimal configurations of the Q and R matrices, with the objective of minimizing the position error with respect to the desired reference.

This work transforms the helicopter position control problem into a computational optimization problem, using the WSO for the tuning of an LQR. The proposed approach is compared with the analytical tuning method and another heuristic alternative, Particle Swarm Optimization (PSO), to determine its effectiveness in controller tuning, as well as its impact on system performance in terms of stability, accuracy and control effort.

2. 2-DOF HELICOPTER

A two-degree-of-freedom (2-DOF) helicopter, as shown in Fig. 1, is a scale model of a real helicopter. This platform consists of a horizontal arm equipped with two rotors, mounted on a vertical base called a pivot. This configuration allows the model to move on two main axes: pitch (θ) and yaw (ψ). The pitch axis corresponds to the raising and lowering of the nose, while the yaw axis allows rotation about the pivot [2].

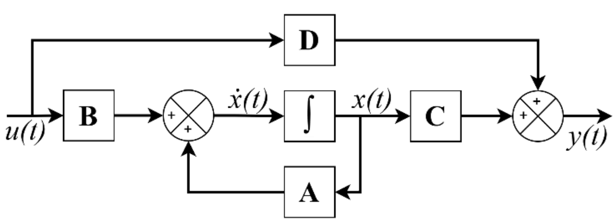
Fig. 1. 2-DOF helicopter free-body diagram.



Source: own elaboration based on [2].

The helicopter is controlled by modulating the power of the rotors, which allows their angular position to be adjusted. To model this dynamic behavior, [2] uses is made of the state space representation shown in Fig. 2 and defined by equations 1 and 2.

Fig. 2. Block diagram of the state space.



Source: own elaboration based on [2].

$$\dot{x}(t) = Ax(t) + Bu(t) \quad [1]$$

$$y(t) = Cx(t) + Du(t) \quad [2]$$

Where $x(t) = [\theta(t), \psi(t), \dot{\theta}(t), \dot{\psi}(t)]$ is the state vector, $u(t) = [u_1(t), u_2(t)]$ is the input vector (voltages applied to the motors) and $y(t) = [\theta(t), \psi(t)]$ is the output vector (pitch and yaw axes). While A, B, C y D are the state model matrices, described in equations 3, 4, 5 and 6.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -9.2768 & 0 \\ 0 & 0 & 0 & -0.34931 \end{bmatrix} \quad [3]$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 2.3667 & 0.0789 \\ 0.2406 & 0.7909 \end{bmatrix} \quad [4]$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad [5]$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad [6]$$

3. LINEAR QUADRATIC REGULATOR

The Linear Quadratic Regulator (LQR) is an optimal control technique that seeks to minimize a cost function that trades off the regulation of the system states and the effort required in the control signals [9]. The design of the LQR is based on finding the control law in equation 7 that minimizes the J function in equation 8.

$$u(t) = -Kx(t) \quad [7]$$

$$J = \int_0^\infty \left(x(t)^T Q x(t) + u(t)^T R u(t)\right) dt \quad [8]$$

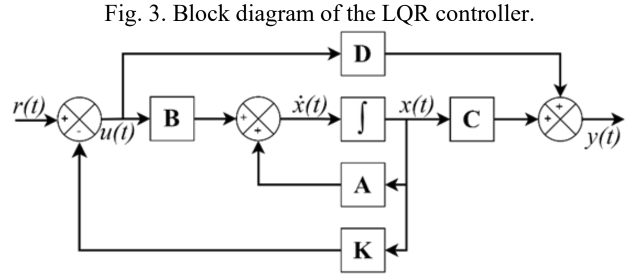
Where Q is a positive semidefinite symmetric matrix penalizing the system states $x(t)$, R is a positive definite symmetric matrix penalizing the control inputs $u(t)$, and K is the state feedback gain matrix shown in 9, calculated using the P matrix resulting from solving the Riccati algebraic equation defined in 10.

$$K = R^{-1} B^T P \quad [9]$$

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad [10]$$

According to [2] and [9], the 2-DOF helicopter attitude control using LQR is illustrated in the diagram in Fig. 3, where the system state vector $x(t)$ is fed back and multiplied by the gain K to generate the control signal $u(t)$. This regulates the pitch and yaw angular positions, trying to make the system reach the desired references.

Fig. 3. Block diagram of the LQR controller.



Source: own elaboration based on [2].

In the context of the 2-DOF helicopter, matrices *Q* and *R,* are specifically defined to capture the system dynamics and meet the control objectives. For the considered model, *Q* is structured as a 4x4 diagonal matrix while *R* is a scalar matrix, as shown in expressions 11 and 12 respectively [2].

$$Q = \begin{bmatrix} q_{11} & 0 & 0 & 0 \\ 0 & q_{22} & 0 & 0 \\ 0 & 0 & q_{33} & 0 \\ 0 & 0 & 0 & q_{44} \end{bmatrix} \quad [11]$$

$$R = [r_{11}] \quad [12]$$

The performance of a system controlled by an LQR depends on the proper choice of the parameters Q and R, which is traditionally done using analytical methods [2][4]. An alternative to this is the use of bio-inspired algorithms, such as the White Shark Optimizer (WSO), which allows the search space of matrices to be automatically explored to identify optimal configurations that maximize system performance.

4. WHITE SHARK OPTIMIZER ALGORITHM

4.1 Population initialization

The White Shark Optimizer is an optimization algorithm bio-inspired by the hunting process of the white shark. It mathematically models the process by which these predators seek, track and capture their prey [11]. To do this, they use an undulating motion along with their acute sense of smell and hearing to locate food sources. They then move towards their target, staying close to the most promising prey and finally, adjusting their movements until they focus on the optimal prey [11][12].

The WSO is performed on a *w* population of *n* sharks, where each individual represents a candidate solution within the search space [11]. The position of each *i-th* shark is denoted by a vector w^i in a *d-th* dimensional space, corresponding to the number of variables to be optimized [11][12]. This is expressed according to [12] in equation 13.

$$w = \begin{bmatrix} w_1^1 & w_2^1 & \cdots & w_d^1 \\ w_1^2 & w_2^2 & & w_d^2 \\ \vdots & & \ddots & \vdots \\ w_1^3 & w_2^3 & \cdots & w_d^n \end{bmatrix} \quad [13]$$

4.2 Exploration

In this first stage, the *w* population of sharks randomly explore the search space, trying to locate potentially promising areas. The position of each w^i shark is calculated using the upper bound *ub* and lower bound *lb* of the search space as shown in 14, where *rand* is a random number in the range of 0-1 [12].

$$w^i = rand \cdot (ub - lb) + lb \quad [14]$$

4.3 Exploitation

Once the most promising prey have been identified, sharks adjust their positions in the direction of the most attractive prey. The speed of this movement is described according to [12] by the following equation:

$$v_{k+1}^i = \mu\left(v_k^i + p_1[w_{gbest_k} - w_k^i] \cdot c_1 + p_2\left[w_{best}^{v_k^i} - w_k^i\right] \cdot c_2\right) \quad [15]$$

where, v_{k+1}^i represents the updated velocity of the *i-th* white shark at iteration $k+1$, v_k^i is the current velocity of the predator at iteration k, w_{gbest_k} denotes the overall best position obtained so far, $w_{best}^{v_k^i}$ represents the best individual position of *i-th* shark and w_k^i is the current location of *i* shark. c_1 y c_2 are random coefficients in the range of [0,1]. Parameters p_1, p_2 and μ are calculated by [13] in equations 16, 17, 18.

$$p_1 = p_{ub} + (p_{ub} - p_{lb}) \cdot e^{-\left(\frac{4k}{k_{max}}\right)^2} \quad [16]$$

$$p_2 = p_{lb} + (p_{ub} - p_{lb}) \cdot e^{-\left(\frac{4k}{k_{max}}\right)^2} \quad [17]$$

$$\mu = \frac{2}{\left|2 - \tau - \sqrt{\tau^2 - 4\tau}\right|}; \quad \tau = 4.125 \quad [18]$$

Accordingly, the position of each shark is updated based on expression 19, such that *mv* is the motion force of the white shark denoted as in equation 20 and grows with each iteration; *a* and *b* represent binary vectors, and *f* is the frequency of the sharks' undulating motion which is expressed in 21 [11][12].

$$w_{k+1}^i = \begin{cases} w_k^i \neg \oplus w_0 + ub \cdot a + lb \cdot b; & rand < mv \\ w_k^i + \dfrac{v_k^i}{f}; & rand \geq mv \end{cases} \quad [19]$$

$$mv = \frac{1}{a_0 + e^{\frac{\frac{k_{max}}{2-k}}{a_1}}}; \quad a_o = 6.25, a_1 = 100 \quad [20]$$

$$f = f_{min} + \frac{f_{max} - f_{min}}{f_{max} + f_{min}}; \quad f_{max} = 0.75, f_{min} = 0.07 \quad [21]$$

4.4 Moving towards the best shark

Finally, sharks update their position towards the best positioned shark to stay close to prey as in equation 22. So $\vec{D}_w$ is the distance between the prey and the shark and is represented as in 23, while s_s describes the strength of the great white shark's senses of sight and smell by expression 24 defined in [12].

$$w_{k+1}^i = w_{gbest_k} + r_1 \vec{D}_w \cdot sgn(r_2 - 0.5); \quad r_3 < s_s \quad [22]$$

$$\vec{D}_w = \left|rand \cdot \left(w_{gbest_k} - w_k^i\right)\right| \quad [23]$$

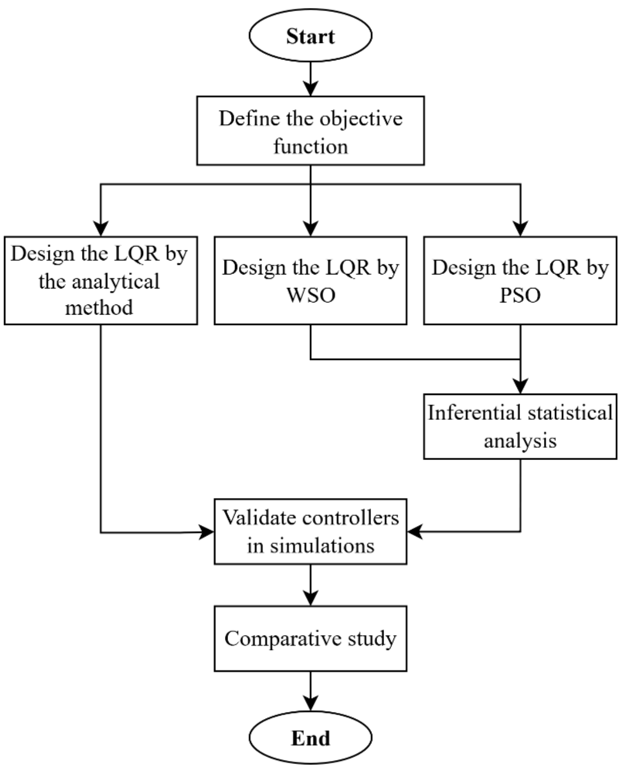
$$s_s = \left|1 - e^{-\frac{0.0005k}{k_{max}}}\right| \quad [24]$$

5. METHODOLOGY

The methodological process followed to evaluate the effectiveness of the WSO algorithm in tuning a LQR for the 2-DOF helicopter is presented in the flow chart in Fig. 4. It begins with the definition of the objective function to be optimized. From this, the WSO and PSO algorithms are implemented for the automatic tuning of the Q and R matrices and, in parallel, an LQR is designed using an analytical method.

To evaluate both algorithms, an inferential statistical analysis is performed, followed by the validation of the three controllers through simulations. Finally, a comparative study is carried out to determine the effectiveness of the WSO in relation to the analytical method and the PSO, allowing its applicability to be evaluated.

Fig. 4. Methodological scheme.



Source: own elaboration.

It is important to note that the objective function to be minimized in this work corresponds to the sum of four performance metrics used in controller analysis: Integral Absolute Error (IAE), Integral Squared Error (ISE), Integral Absolute Error in Time (ITAE) and Integral Squared Error in Time (ITSE). These metrics allow the evaluation of the helicopter's behavior in terms of precision, stability and response time. The mathematical expression of this is defined in 25, solving it as a single-objective problem.

$$F_{obj}(\vec{x}) = IAE+ISE+ITAE+ITSE \quad [25]$$

The performance indices are described in equations 26, 27, 28 and 29. Here, *e(t)* represents the system error, defined as the difference between the desired position reference and the actual position of the system [9].

$$IAE = \int_0^T |e(t)|dt \quad [26]$$

$$ISE = \int_0^T e^2(t)dt \quad [27]$$

$$ITAE = \int_0^T t \cdot |e(t)|dt \quad [28]$$

$$ITSE = \int_0^T t \cdot e^2(t)dt \quad [29]$$

Table 1 presents the configuration parameters used in the design of the WSO and PSO algorithms for LQR controller tuning. Both algorithms optimize the same set of variables, corresponding to the elements of the matrices Q and R, within a defined search space. A fixed number of iterations and agents are used to ensure a fair comparison between both heuristics.

Table 1. Design parameters of the algorithms.

Parameter	WSO	PSO
Search space	[0, 100]	[0, 100]
Variables	$[q_{11}, q_{22}, q_{33}, q_{44}, r_{11}]$	$[q_{11}, q_{22}, q_{33}, q_{44}, r_{11}]$
Iterations	60	60
Number of agents	100	100

Source: own elaboration.

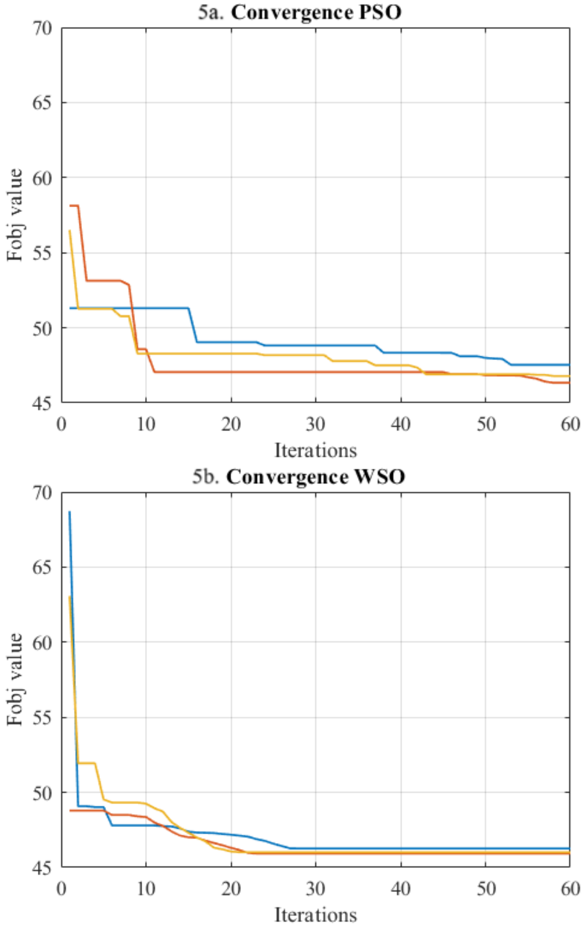
6. RESULTS

The first evaluation of the optimization algorithms' performance is based on their ability to minimize the defined objective function. Fig. 5 shows an example of the minimization process, comparing the convergence of the PSO (Fig. 5a) and WSO (Fig. 5b) in a representative run, showing the optimization trajectories for the smallest, largest, and average values of the objective function in each case.

The analysis of the convergence curves presented shows significant differences in the behavior of the algorithms. The PSO converges more slowly, reaching the optimal value between iterations 50 and 60, with greater variability in the result. In contrast, the WSO shows faster convergence, reducing the objective function value in fewer than 30 iterations and with less variability between runs, making it a better alternative in terms of speed and stability.

The second part of the evaluation consists of the inferential statistical analysis, in which the results obtained by both heuristics are examined after 40 independent executions. Initially, the Kolmogorov-Smirnov test is used, which indicates that the two algorithms do not follow a normal distribution. Therefore, a Friedman test is applied, which indicates that there are statistically significant differences between the two approaches.

Fig. 5. Convergence comparison.



Source: own elaboration.

Due to the non-normality of the data, the median is used as a representative measure of the value of the objective function obtained by each algorithm. Additionally, the 1st and 3rd quartiles are calculated to obtain the interquartile range (IQR) as a dispersion metric of the results. Table 2 summarizes the values obtained in the statistical tests for both algorithms.

Table 2. Statistical analysis.

Parameter	PSO	WSO
Kolmogórov-Smirnov test	$1.0843x10^{-36}$	$1.4002x10^{-36}$
Friedman test	$2.1014\ x10^{-6}$	
Median	46.7870	46.0050
Quartile 1	46.3650	45.9250
Quartile 3	47.5325	46.2360
IQR	1.1675	0.3110

Source: own elaboration.

The values show that the WSO achieves a lower median of the objective function, indicating a better performance in the optimization. Also, its interquartile range is smaller, reflecting a lower variability and, therefore, greater stability in the results obtained, which makes it a more consistent alternative for controller optimization, compared to the PSO.

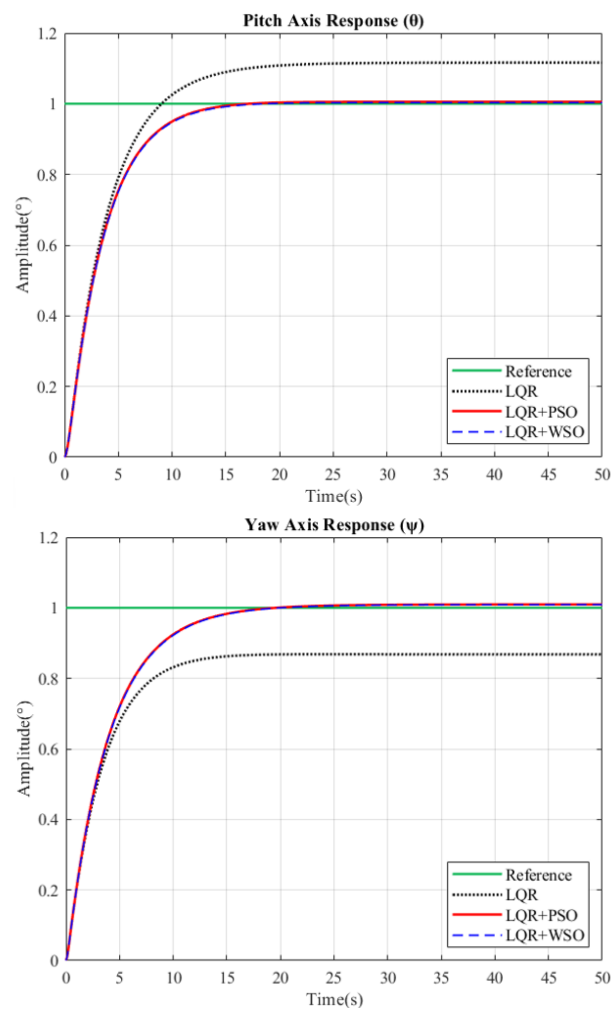
Table 3 presents the Q and R matrices resulting from the WSO and PSO produced by the median of the best agents in the 40 executions performed above. These parameters are subjected to validation by simulating the LQR controller in the adjustment of the helicopter's position. The responses of the pitch (θ) and yaw (ψ) angles to a step-type reference signal are shown in Fig. 6, their performance indicators are summarized in Table 4 and the best ones are highlighted.

Table 3. Tuned arrays.

	Q	R
PSO	$\begin{bmatrix} 53.35 & 0 & 0 & 0 \\ 0 & 98.95 & 0 & 0 \\ 0 & 0 & 13.67 & 0 \\ 0 & 0 & 0 & 16.99 \end{bmatrix}$	[77.20]
WSO	$\begin{bmatrix} 45.90 & 0 & 0 & 0 \\ 0 & 85.34 & 0 & 0 \\ 0 & 0 & 23.19 & 0 \\ 0 & 0 & 0 & 28.04 \end{bmatrix}$	[66.30]

Source: own elaboration.

Fig. 6. Performance comparison of the proposed controllers in both the pitch and yaw axes of the helicopter.



Source: own elaboration.

Table 4. Comparison of performance indicators.

		Settling time	Steady state error	IAE	ISE	ITAE	ITSE
LQR	θ	18s	-12%	7.39	2.32	141.94	18.02
	ψ	20s	13%	9.44	2.92	173.52	26.11
LQR+PSO	θ	17s	0.86%	3.95	2.03	15.70	3.94
	ψ	19s	0.54%	3.61	1.99	12.20	3.36
LQR+WSO	θ	17s	0.79%	3.93	2.01	15.50	3.90
	ψ	19s	0.32%	3.59	1.96	12.07	3.04

Source: own elaboration.

7. CONCLUSIONS AND RECOMMENDATIONS

The results of this work demonstrate the feasibility of using Artificial Intelligence in the tuning of the LQR controller applied to the 2-DOF helicopter. Both WSO and PSO significantly improved the position adjustment compared to the traditional analytical design. The statistical analysis showed that WSO achieved a greater reduction in the system error rates and presented a lower variability in its results compared to PSO, highlighting its stability and precision in controller optimization.

On the other hand, in the evaluation of the controller performance, both heuristic algorithms managed to reduce the steady-state error and improve the transient response compared to LQR without optimization. Although the differences between WSO and PSO are minimal, WSO presented a lower steady-state error, as well as a better frightening of the performance indicators, suggesting a greater effectiveness of the controller in adjusting the helicopter's position.

As future work, a comparison of the approaches implemented in this work with other bio-inspired optimization heuristics is proposed, in order to evaluate their performance in LQR tuning. Furthermore, the use of other control schemes will be explored, with the aim of improving the helicopter response in terms of settling time, steady-state error, and performance metrics.

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